HEAVY BOSONS, REGGE TRAJECTORIES AND DYNAMICAL THEORIES

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It is argued that the narrowness of the higher boson resonances, R, S, T and U, is incompatible with the bootstrap hypothesis if these resonances are regarded as lying on straight Regge trajectories passing through the ρ and A_2.

The purpose of this note is to point out that the supposition that the boson resonances discovered by Foccaci et al. [1] lie on two approximately straight Regge trajectories is incompatible with the bootstrap hypothesis.

It has often been noted that the continuation upward of the Regge trajectories for a large range of energy, and the approximate linearity of the trajectories (see fig. 1), is not consistent with a bootstrap model based on elastic unitarity, since Levinson's theorem constrains the trajectories to turn over. However, it is easy to imagine that as we increase the energy the trajectories, $\alpha(s)$, may become strongly coupled to the new channels which open up, and so continue to rise [e.g. 2]. If this behaviour were to continue indefinitely so that $\alpha(s) \sim s$ as $s \to \infty$, then the amplitude ($\sim t^{\alpha(s)}$) would not be uniformly power bounded in t, and so the Mandelstam representation would not hold. Even in this situation one could hope that, by including only a few low mass channels, one could obtain a reasonable approximation at least for the physically interesting region of a trajectory, that is for s adjacent to zero.

We wish to show that the behaviour of the widths of resonances found by Foccaci et al. [1] makes such a hope untenable. To this end we follow Spector [3] (though with a very different interpreation of the data) in using the relation between the total width of a resonance, Γ, and $\mathrm{Im}\{\alpha(s)\}$ which is given by Newton [4];

$$\Gamma = \frac{\mathrm{Im}\,\alpha(s)\,\mathrm{Re}\,\alpha'(s)s^{-\frac{1}{2}}}{[\mathrm{Re}\,\alpha'(s)]^2 + [\mathrm{Im}\,\alpha'(s)]^2} \qquad (1)$$

the derivatives being taken at the value of s such that $\mathrm{Re}\{\alpha(s)\}$ is equal to the spin of the resonance. Provided $\mathrm{Im}\,\alpha'$ is not greatly in excess of $\mathrm{Re}\,\alpha'$ we find *

$$\mathrm{Im}\{\alpha(s)\} \approx \Gamma\ \mathrm{Re}\,\alpha'(s)s^{\frac{1}{2}} \qquad (2)$$

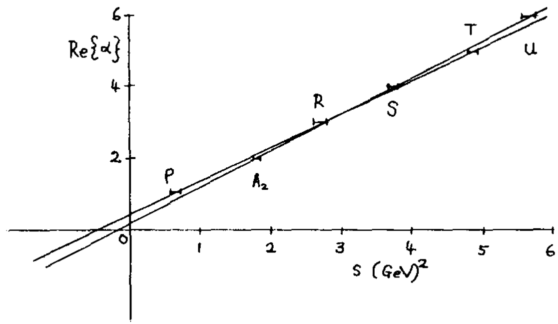

Fig. 1. The boson resonances interpreted as lying on straight Regge trajectories. The straight lines are
$\alpha_{\rho}(s) = 0.43 + 0.948\ s$
$\alpha_{A_2}(s) = 0.26 + 0.011\ s$.

We consider the ρ and A_2 trajectories, on which lie, according to the hypothesis of straight Regge trajectories, the ρ, R and T, and the A_2, S and U resonances, respectively; see fig. 1. (We do not assume exchange degeneracy. If we did we could include all these resonances on the same trajectory, and the argument would be somewhat strengthened.) From eq. (2), using the values for the widths given by Rosenfeld et al. [5], we derive the values for $\mathrm{Im}\{\alpha(s)\}$ given in fig. 2. The important point to note is that the higher mass states are very narrow, so $\mathrm{Im}\{\alpha\}$ must be small at high energy. This would not be the case if the trajectories were strongly coupled to higher threshold channels.

We assume a conventional dispersion relation for α, with only a right-hand cut. On this unitarity constrains $\mathrm{Im}\{\alpha\}$ to be positive. It is well known that if two trajectories cross they can also develop

* We ignore at present the bizarre possibility that $\mathrm{Im}\,\alpha' \gg \mathrm{Re}\,\alpha'$, which from eq. (1) would allow larger values of $\mathrm{Im}\,\alpha$ for a given Γ, but which is apparently incompatible with a dispersion relation for α.

left-hand cuts. However, these can not play an important role in the dynamics since the sum of their discontinuities has to vanish. It would simply mean that we should have to include both trajectories in our discussion. We shall ignore this possibility from now on. If we assume that $\alpha(s) \to a$ (a = constant) and Im $\alpha \to 0$ like some power of s as $s \to \infty$ we can write

$$\alpha(s) = a + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im}\{\alpha(s')\}}{s' - s} \mathrm{d}s' , \qquad (3)$$

where s_0 is the threshold.

A reasonable fit to Im$\{\alpha(s)\}$ is obtained with the parameterization of Ahmadzadeh and Sakmar [6]

$$\mathrm{Im}\{\alpha(s)\} = \frac{c(s - s_0)^{\lambda}}{(s - d)^2 + e^2} , \qquad (4)$$

with $\lambda = 0.0001$, $d = 2.0$ (GeV)^2, $e = 1$ (GeV)^2 and $c = 0.31$ for the ρ trajectory and 0.145 for the A_2. Note that Im$\{\alpha(s)\}$ must be zero at threshold, a fact not used by Spector [3]. The slope of the trajectory is not very sensitive to the choice of λ. The trajectories obtained from eq. (3), with $a = 0$, are shown in fig. 2, and we see that the contribution of the integral is negligible. So although with a suitable choice of a we can make a trajectory pass through any given point, there is no hope of obtaining the correct slope.

We now consider what possibilities exist for solving this difficulty. First we note that is does not help to change the form of Im$\{\alpha(s)\}$ so that it tails off slowly, i. e. like $\sim s^{-\epsilon}$ where ϵ is small and positive. This would allow us to make the contribution of the integral as large as we like, but the contribution to $\mathrm{d}\alpha/\mathrm{d}s$ for small s would still be negligible.

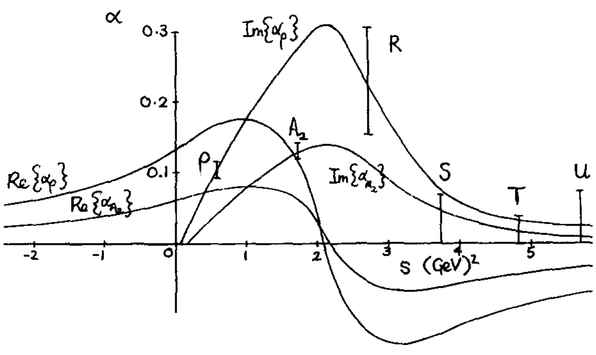

Fig. 2. The values of Im $\{\alpha(s)\}$ obtained from eq. (2), and fitted by eq. (4) for the ρ and A2 trajectories. The real parts of $\alpha(s)$ are obtained from eq. (3) with a taken arbitrarily to be zero.

We appear to be faced with three alternatives.

(a) Eq. (3) for $\alpha(s)$ requires an additional subtraction

$$\alpha(s) \doteq a + bs + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im}\{\alpha(s')\}}{s' - s} \mathrm{d}s' . \qquad (5)$$

The arbitrary subtraction constant, b, is responsible for the slope of the trajectories in the low energy region. This seems to us a very unpalatable suggestion as it implies that, unlike potential scattering, the "forces" (the double spectral functions) do not determine the important parameters of the trajectories, and hence are not responsible for the existence of the particles *. This would kill the whole bootstrap philosophy, which depends on the assumption that the double-spectral-functions determine the dynamics at least in the low energy region **.

(b) The curve for Im$\{\alpha(s)\}$ rises again at higher energies. This would seem to imply that there was some other mass "scale" in the problem. This would be the case if, for example, the mesons were made up of very heavy particles, e.g. "quarks" (though of course the SU(3) properties of conventional quarks are irrelevant here). It is then found that if we assume that the trajectories are (at least to a good approximation) bound states of these quarks (i. e. of a q-$\bar{\mathrm{q}}$ pair), then the straightness of the trajectories is inevitable, *and independent of any further assumptions about the q-$\bar{q}$ potential.* To see this we write a dispersion relation like eq. (3)

$$\alpha(s) = \alpha(\infty) + \frac{1}{\pi} \int_{s_T}^{\infty} \frac{\mathrm{Im}\{\alpha(s')\}}{s' - s} \mathrm{d}s' , \qquad (6)$$

where now the integral begins at s_T, the q-$\bar{\mathrm{q}}$ threshold, and we are assuming that it converges. Since s_T is very large, and Im$\{\alpha\}$ has to be positive, we can approximate the integral by a δ-function at $s = s_p > s_T$ if we are only concerned with $s \ll s_T$. Thus

* Note that although the discontinuity behaves as $t^{\alpha(s)}$ for large t, the double-spectral-function behaves as $t\mathrm{Im}\,\alpha(s)$ and the integral over the strip region of the double-spectral-function is insufficient to give the required trajectory no matter how wide the strip is taken.

** This problem is not circumvented by the use of finite-energy-sum rules to apply crossing as in the method of Mandelstam [7]. If one attempted a complete treatment of the dynamics using the Cheng and Sharp [8] equations then one would again find the necessity of inserting the essential property of the trajectory as an arbitrary constant.

$$\alpha(s) = \alpha(\infty) + \frac{g}{s_p - s} \qquad (s \ll s_p, s_p > s_T) , \qquad (7)$$

which for small s can be written

$$\alpha(s) = \alpha(\infty) + \frac{g}{s_p} + \frac{g}{s_p}\left(\frac{s}{s_p}\right) + \frac{g}{s_p}\left(\frac{s}{s_p}\right)^2 + \ldots \qquad (8)$$

Since the trajectory passes through (say) the ρ we must have

$$1 = \alpha(\infty) + \frac{g}{s_p - m_\rho^2} , \qquad (9)$$

so that

$$\alpha(\infty) \approx 1 - g/s_p . \qquad (10)$$

The slope of this trajectory at $s = 0$ is (g/s_p^2), which according to fig. 1 is about $1(\mathrm{GeV})^{-2}$, so eq. (10) gives

$$\alpha(\infty) \approx 1 - s_p/(1\ \mathrm{GeV})^2 . \qquad (11)$$

Hence the trajectory must pass through many $[\sim s_p/(1\ \mathrm{GeV})^2]$ negative integers, which requires that $q\bar{q}$ potential be such that a large number of superconvergence relations are satisfied. There is no obvious objection to this as we know that similar superconvergence relations must also obtain when the particles involved in the scattering process have spin [9], but it means that q-$\bar{q}$ potential must be quite unlike the simple forms which have sometimes been used in the literature, and which force $\alpha(\infty) = -1$. Note that the quadratic and higher terms in [8] are negligible in the physically interesting region.

(c) The interpretation of the higher mass bosons as Regge recurrences of the lower ones is wrong. This solution will be confirmed or refuted when the spins of the higher boson resonances are determined. We hope this note will add further encouragement to those who are attempting to determine these spins.

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